

Divisibility Rules

Div. by 2: Check the last digit
0, 2, 4, 6, 8

Div. by 5: Check the last digit
0, 5

Div. by 3: abc (mod 3)

$$\underline{abc} = 100a + 10b + c \pmod{3}$$

$$\equiv \cancel{99}a + a + \cancel{9}b + b + c \pmod{3}$$

$$\equiv a + b + c \pmod{3}$$

$$321 \equiv 3 + 2 + 1 \equiv 6 \pmod{3}$$

$$12345 \equiv 1 + 2 + 3 + 4 + 5 \equiv 15 \pmod{3}$$

$$952 \equiv 9 + 5 + 2 \equiv 16 \equiv 1 \pmod{3}$$

Div. by 9: abc mod 9

$$\underline{abc} = 100a + 10b + c \pmod{9}$$

$$\equiv \cancel{90}a + a + \cancel{9}b + b + c \pmod{9}$$

$$\equiv a + b + c \pmod{9}$$

$$1234 \equiv 1 + 2 + 3 + 4 \equiv 10 \equiv 1 \pmod{9}$$

Div. by 2^n : Check the last n digits

$$\underline{4096} \text{ div. by } 4?$$

$$96 = 4 \cdot 24 \quad \checkmark$$

$$\underline{4096} \text{ div. by } 8?$$

$$96 = 8 \cdot 12 \quad \checkmark$$

Div. by 5^n : Check the last n digits

$$\underline{375} \text{ div. by } 5^2 = 25?$$

$$\underline{75} = 25 \cdot 3 \quad \checkmark$$

Div. by 6: Check div. by 2
check div. by 3

$$1125 = 9 \cdot 125 = \underbrace{3^2} \cdot \underbrace{5^3}$$

Div. by 11: abcd mod 11

$$10 \equiv -1 \pmod{11}$$

$$\underline{abcd} \equiv 1000a + 100b + 10c + d \pmod{11}$$

$$\equiv -a + b - c + d \pmod{11}$$

$$12345 \equiv +1 - 2 + 3 - 4 + 5 \equiv 3 \pmod{11}$$

$$9876 \equiv -9 + 8 - 7 + 6 \equiv -2 \equiv 9 \pmod{11}$$