

The Ninth Grade Math Competition Class
Quadratic Formula and Polynomial
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1. Find the value of x if x is positive and $x - 1$ is the reciprocal of $x + \frac{1}{2}$.

$$x - 1 = \frac{1}{x + \frac{1}{2}}$$

$$(x - 1)\left(x + \frac{1}{2}\right) = 1$$

$$x^2 - x + \frac{1}{2}x - \frac{1}{2} = 1$$

$$x^2 - \frac{1}{2}x - \frac{3}{2} = 0$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$x = \frac{3}{2}$ ~~$x = -1$~~

2. It is given that one root of $2x^2 + rx + s = 0$, with r and s real numbers, is $3 + 2i$. Find s .

$$3 + 2i$$

$$3 - 2i$$

$$(3 + 2i)(3 - 2i) = \frac{s}{2}$$

$$13 = \frac{s}{2} \Rightarrow s = 26$$

3. Find all values of k such that $x^2 + kx + 27 = 0$ has two distinct real solutions for x .

$$k^2 - 4 \cdot 27 > 0$$

$$k^2 > 108$$

$$k > \sqrt{108} = 6\sqrt{3}$$

$$k < -\sqrt{108} = -6\sqrt{3}$$

4. Find all real solutions to $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$.

$$1^a = 1$$

$$a^0 = 1$$

even

$$-1^a = 1$$

$$x^2 - 5x + 5 = 1$$

$$x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) \Rightarrow$$

$$x=1, 4$$

$$x^2 - 9x + 20 = 0$$

$$(x-4)(x-5) \Rightarrow$$

$$x=4, 5$$

$$x^2 - 5x + 5 = -1$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) \Rightarrow$$

$$x=2, 3$$

5. Find all real solutions (x, y) of the system $x^2 + y = 12 = y^2 + x$.

$$\begin{aligned}
 x^2 + y &= 12 \\
 y^2 + x &= 12 \\
 x^2 + y &= y^2 + x \\
 x^2 - y^2 &= x - y \\
 (x - y)(x + y) &= x - y \\
 \text{or } x + y &= 1 \\
 y &= 1 - x \\
 x^2 + 1 - x &= 12 \\
 x^2 - x - 11 &= 0 \\
 \frac{1 \pm \sqrt{1 + 44}}{2} &= \frac{1 \pm \frac{\sqrt{45}}{2}}{2} = \frac{1}{2} \pm \frac{3\sqrt{5}}{2}
 \end{aligned}$$

$\Rightarrow x = y$

$$\begin{aligned}
 x^2 + x &= 12 \\
 x^2 + x - 12 &= 0 \\
 \frac{-1 \pm \sqrt{1 + 48}}{2} \\
 \frac{-1 \pm 7}{2} \\
 x &= -3, -4
 \end{aligned}$$

$$\left(\frac{1}{2} + \frac{3\sqrt{5}}{2}, \frac{1}{2} - \frac{3\sqrt{5}}{2} \right) \\
 \left(\frac{1}{2} - \frac{3\sqrt{5}}{2}, \frac{1}{2} + \frac{3\sqrt{5}}{2} \right)$$

6. Find all values of m for which the zeros of $2x^2 - mx - 8$ differ by $m - 1$.

P(x)

7. A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

(A) $\frac{1+i\sqrt{11}}{2}$ (B) $\frac{1+i}{2}$ (C) $\frac{1}{2} + i$ (D) $1 + \frac{i}{2}$ (E) $\frac{1+i\sqrt{13}}{2}$

(B) $r = \frac{1}{2} + \frac{i}{2}$

$s = \frac{1}{2} - \frac{i}{2}$

r, s, t, u

$P(x) = (x-r)(x-s)(x-t)(x-u)$

$P(x) = \left(x - \frac{1}{2} - \frac{i}{2}\right) \left(x - \frac{1}{2} + \frac{i}{2}\right) (x-t)(x-u)$

$\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$

$x^2 - x + \frac{1}{4} + \frac{1}{4}$

$\left(x^2 - x + \frac{1}{2}\right) (x-t)(x-u)$

(A) $r = \frac{1}{2} + \frac{\sqrt{11}i}{2}$

$s = \frac{1}{2} - \frac{\sqrt{11}i}{2}$

$P(x) = \left(x - \frac{1}{2} - \frac{\sqrt{11}i}{2}\right) \left(x - \frac{1}{2} + \frac{\sqrt{11}i}{2}\right) (x-t)(x-u)$
 $= \left(x - \frac{1}{2}\right)^2 + \frac{11}{4}$
 $= \left(x^2 - x + \frac{1}{4} + \frac{11}{4}\right) (x-t)(x-u)$
 $= \left(x^2 - x + \frac{12}{4}\right) (x-t)(x-u)$
 $= \left(x^2 - x + 3\right) (x-t)(x-u)$

8. Find the sum of all the roots of the equation $x^{2001} + (\frac{1}{2} - x)^{2001} = 0$.

9. Three of the roots of $x^4 + ax^2 + bx + c = 0$ are -2 , -3 , 5 . Find the value of $a + b + c$.

10. One root of the quadratic $x^2 + bx + c = 0$ is $1 - 3i$. If b and c are real numbers, then what are b and c ?

$$1 - 3i$$

$$1 + 3i$$

$$1 - 3i + 1 + 3i = -\frac{b}{1}$$

$$\Rightarrow 2 = -b$$

$$b = 2$$

$$(1 - 3i)(1 + 3i) = \frac{c}{1}$$

$$1 - 3i + 3i + 9 = \frac{c}{1}$$

$$\Rightarrow c = 10$$

- 11.** Suppose the roots of $x^3 + 3x^2 + 4x - 11 = 0$ are a , b and c , and the roots of $x^3 + rx^2 + sx + t = 0$ are $a + b$, $b + c$, and $c + a$, find the value of t .

12. Let a , b , and c be the roots of $x^3 - 3x^2 + 1$.

- Find a polynomial whose roots are $a + 3$, $b + 3$ and $c + 3$.
- Find a polynomial whose roots are $\frac{1}{a+3}$, $\frac{1}{b+3}$, and $\frac{1}{c+3}$.
- Compute $\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}$.
- Find a polynomial whose roots are a^2 , b^2 and c^2 .

13. The equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$ has three real roots. Find their sum.

- 14.** If $P(x)$ is a polynomial in x such that for all x , $x^{23} + 23x^{17} - 18x^{16} - 24x^{15} + 108x^{14} = (x^4 - 3x^2 - 2x + 9) \cdot P(x)$, compute the sum of coefficients of $P(x)$.

15. The real number x satisfies the equation $x + \frac{1}{x} = \sqrt{5}$. What is the value of $x^{11} - 7x^7 + x^3$?

- 16.** All the roots of the polynomial $x^6 - 10z^5 + Az^4 + Bz^3 + cZ^2 + Dz + 16$ are positive integers, possibly repeated. What is the value of B ?