Divisibility Rules

Divisibility by 5: does it end in 0 or 5?

$10a + 5$

Div. by 2: is the units digit even?

$10a + b$

Div. by 3: $2049 = 2 \cdot 10^3 + 0 \cdot 10^2 + 4 \cdot 10^1 + 9$

$2 + 0 + 4 + 9 = 15$

$\equiv 2 \cdot 1^3 + 0 \cdot 1^2 + 4 \cdot 1^1 + 9$

$\equiv 2 + 0 + 4 + 9 \pmod 3$

Div. by 9: $2049 = 2 \cdot 10^3 + 0 \cdot 10^2 + 4 \cdot 10^1 + 9$

$2 + 0 + 4 + 9 = 15$

$\equiv 2 \cdot 1^3 + 0 \cdot 1^2 + 4 \cdot 1^1 + 9$

$\equiv 2 + 0 + 4 + 9 \pmod 9$

$81 = 27 \cdot 3$

$8 + 1$

Div. by 4: $2048 = 20 \cdot 100 + 48$

Div. by 2^n = a 10^nth: last n digits divisible by 2^n

Div. by 25: last 2 ... div. by 25

5^n: last n ... div. by 5^n

Div. by 6 $\quad 6 = 2^1 \cdot 3^1$

Div. by 90 $\quad 90 = 2^1 \cdot 3^2 \cdot 5^1$

Div. by 11 $\quad \underline{ABCD} = 10^3 \cdot A + 10^2 \cdot B + 10^1 \cdot C + D$

$\equiv (-1)^3 \cdot A + (-1)^2 \cdot B + (-1)^1 \cdot C + D \pmod{11}$

$\equiv -A + B - C + D \pmod{11}$

$1234 \equiv -1 + 2 - 3 + 4 \equiv -2 \equiv 9 \pmod{11}$