

The Ninth Grade Math Competition Class
Quadratic Equations and Vieta
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1. Let a and b denote the solutions of $18x^2 + 3x - 28 = 0$, find the value of $(a - 1)(b - 1)$.

2. Let a and b be the roots of the equation $x^2 - mx + 2 = 0$. Suppose that $a + \frac{1}{b}$ and $b + \frac{1}{a}$ are roots of the equation $x^2 - px + q = 0$, find q .

- 3.** Let p , q and r be constants. One solution to the equation $(x - p)(x - q) = (r - p)(r - q)$ is $x = r$. Find the other solution in terms of p , q and r .

4. If m and n are the roots of $x^2 + mx + n = 0$, where $m \neq 0$ and $n \neq 0$, then what number does $m + n$ equal?

5. For what values of k does the equation $\frac{x-1}{x-2} = \frac{x-k}{x-6}$ have no solution for x ?

6. Find all solutions to $2w^4 - 5w^2 + 2 = 0$.

7. Find the value of $\sqrt{90 + \sqrt{90 + \sqrt{90 + \cdots}}}$.

8. Let a and b be the roots of $x^2 - 3x - 1 = 0$. Try to solve the following problems without finding a and b , it will be easier that way, anyway.
- Find a quadratic equations whose roots are a^2 and b^2 .
 - Compute $\frac{1}{a+1} + \frac{1}{b+1}$.

9. For some integer a , the equation $1988x^2 + ax + 8891 = 0$, and $8891x^2 + ax + 1988 = 0$ share a common root. Find a .

10. The product of the roots of the quadratic $6x^2 + cx + 4$ is 2 greater than the sum of the roots, and c is a constant. What is c ?

11. Let $a, b,$ and c be the roots of $x^3 - 3x^2 + 1$.

- Find a polynomial whose roots are $a + 3, b + 3$ and $c + 3$.
- Find a polynomial whose roots are $\frac{1}{a+3}, \frac{1}{b+3},$ and $\frac{1}{c+3}$.
- Compute $\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}$.
- Find a polynomial whose roots are a^2, b^2 and c^2 .
- Find a recurrence relation for $x_n = a^n + b^n + c^n,$ and use it to compute $a^5 + b^5 + c^5$.