

The Ninth Grade Math Competition Class
Quadratic Formula and Polynomial
Anthony Wang

1. Find the value of x if x is positive and $x - 1$ is the reciprocal of $x + \frac{1}{2}$.

2. It is given that one root of $2x^2 + rx + s = 0$, with r and s real numbers, is $3 + 2i$. Find s .

3. Find all values of k such that $x^2 + kx + 27 = 0$ has two distinct real solutions for x .

4. Find all real solutions to $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$.

5. Find all real solutions (x, y) of the system $x^2 + y = 12 = y^2 + x$.

6. Find all values of m for which the zeros of $2x^2 - mx - 8$ differ by $m - 1$.

7. A polynomial of degree four with leading coefficient 1 and integer coefficients has two zeros, both of which are integers. Which of the following can also be a zero of the polynomial?

(A) $\frac{1+i\sqrt{11}}{2}$ (B) $\frac{1+i}{2}$ (C) $\frac{1}{2} + i$ (D) $1 + \frac{i}{2}$ (E) $\frac{1+i\sqrt{13}}{2}$

8. Find the sum of all the roots of the equation $x^{2001} + (\frac{1}{2} - x)^{2001} = 0$.

9. Three of the roots of $x^4 + ax^2 + bx + c = 0$ are -2 , -3 , 5 . Find the value of $a + b + c$.

- 10.** One root of the quadratic $x^2 + bx + c = 0$ is $1 - 3i$. If b and c are real numbers, then what are b and c ?

11. Suppose the roots of $x^3 + 3x^2 + 4x - 11 = 0$ are a , b and c , and the roots of $x^3 + rx^2 + sx + t = 0$ are $a + b$, $b + c$, and $c + a$, find the value of t .

12. Let a , b , and c be the roots of $x^3 - 3x^2 + 1$.

- Find a polynomial whose roots are $a + 3$, $b + 3$ and $c + 3$.
- Find a polynomial whose roots are $\frac{1}{a+3}$, $\frac{1}{b+3}$, and $\frac{1}{c+3}$.
- Compute $\frac{1}{a+3} + \frac{1}{b+3} + \frac{1}{c+3}$.
- Find a polynomial whose roots are a^2 , b^2 and c^2 .

13. The equation $2^{333x-2} + 2^{111x+2} = 2^{222x+1} + 1$ has three real roots. Find their sum.

- 14.** If $P(x)$ is a polynomial in x such that for all x , $x^{23} + 23x^{17} - 18x^{16} - 24x^{15} + 108x^{14} = (x^4 - 3x^2 - 2x + 9) \cdot P(x)$, compute the sum of coefficients of $P(x)$.

15. The real number x satisfies the equation $x + \frac{1}{x} = \sqrt{5}$. What is the value of $x^{11} - 7x^7 + x^3$?

16. All the roots of the polynomial $x^6 - 10z^5 + Az^4 + Bz^3 + cZ^2 + Dz + 16$ are positive integers, possibly repeated. What is the value of B ?