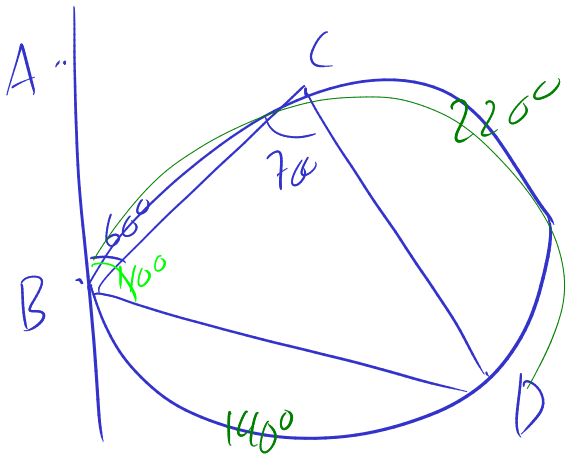
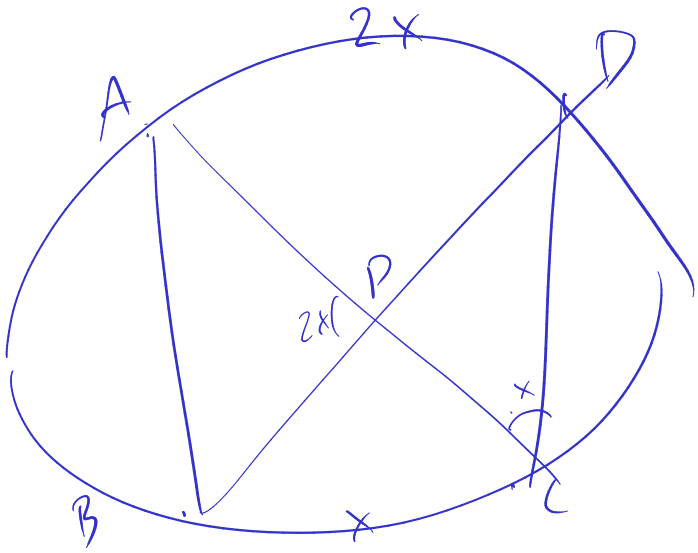
The Ninth Grade Math Competition Class
Angles, Arcs, and Special Trianles
Anthony Wang

1. In the figure, given that $\angle ABC = 60°$, and $\angle BCD = 70°$, find $\angle CBD$.



$= 50°$

2. Find x such that $\angle APB = 2x$, $\angle ACD = x$ and $\overset{\frown}{BC} = x$.

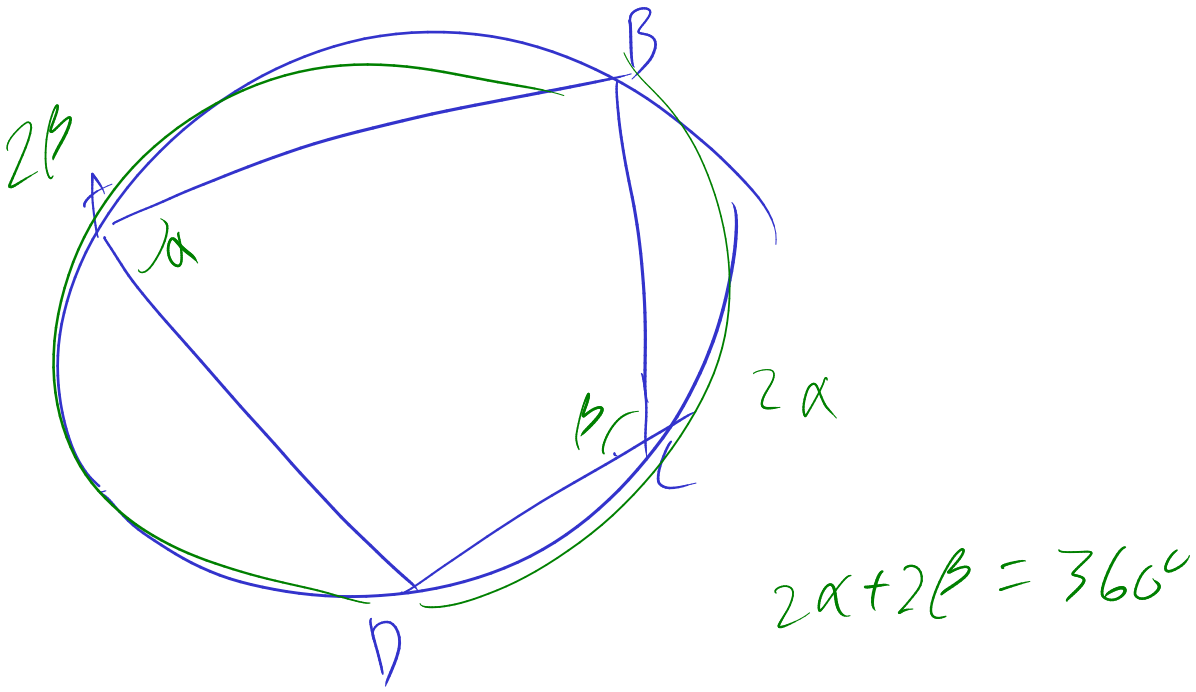


$$\frac{\overset{\frown}{AB} + \overset{\frown}{DC}}{2} = 2x$$

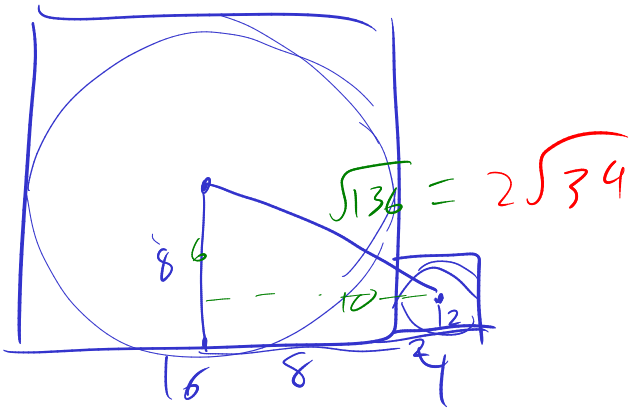
$$4x + 2x + x = 360$$

$$x = \frac{360}{7}$$

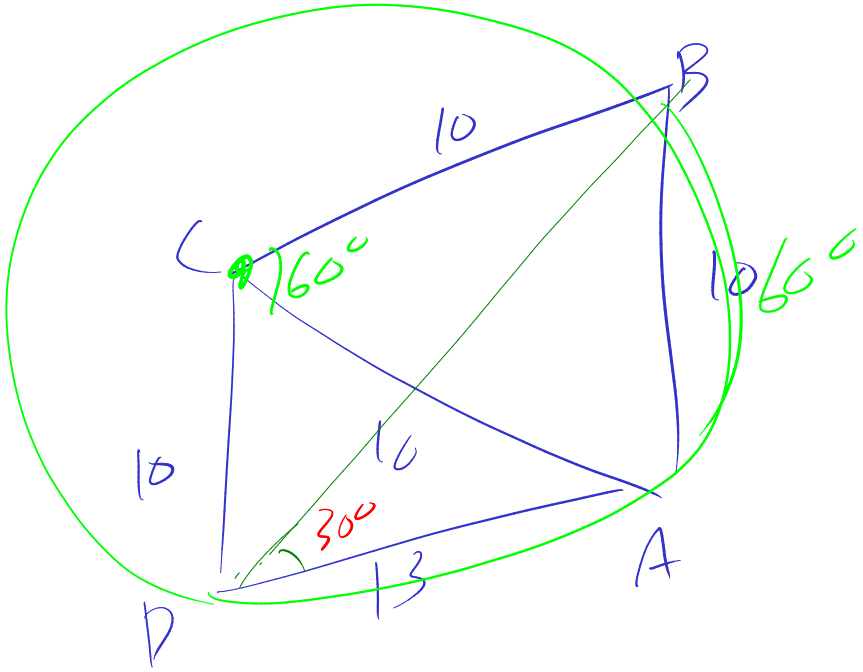
3. A quadrilateral is said to be cyclic quadrilateral if a circle can be drawn that passes through all four of its vertices. Prove that if ABCD is a cycli quadrilateral, then $\angle A + \angle C = 180°$. Such a quadrilateral is said to be inscribed in the circle.



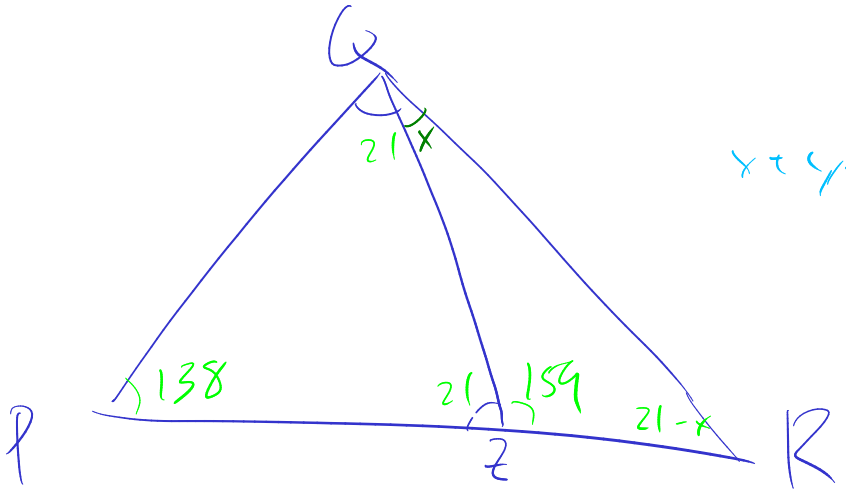
4. The areas of two adjacent squares are 256 square inches and 16 square inches, respectively, and their bases are on the same line. What is the number of inches in the length of the segment that joins the center of the two inscribed circles?



5. We are given points A, B, C, D in the plane such that $AD = 13$, $AB = BC = AC = CD = 10$, find $\angle ADB$.



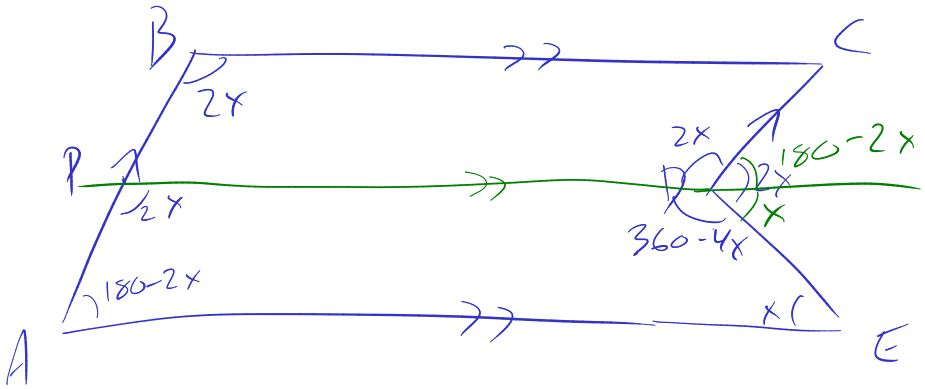
6. Point Z is on side PR of $\triangle PQR$ such that $\angle PZQ = \angle PQZ$ and $\angle PQR - \angle PRQ = 42°$, find $\angle RQZ$.



$x + y - (y - x) = 42$

$2x = 42 \quad x = 21$

7. See the figure above and find the value of X.

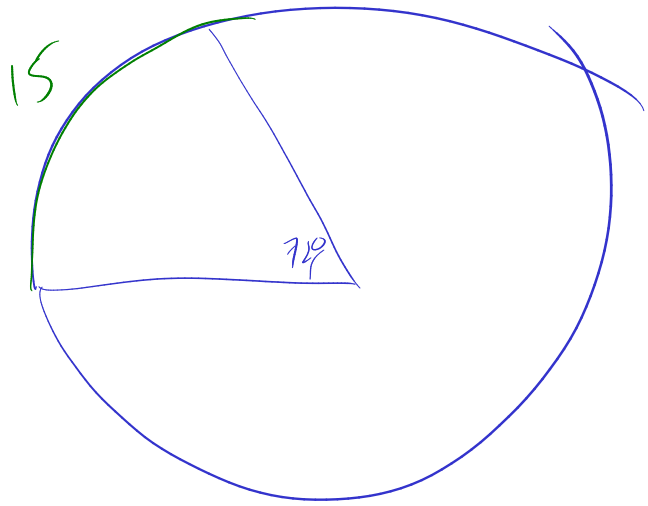


$x + 180 - 2x = 2x$

$180 = 5x$

$x = 36$

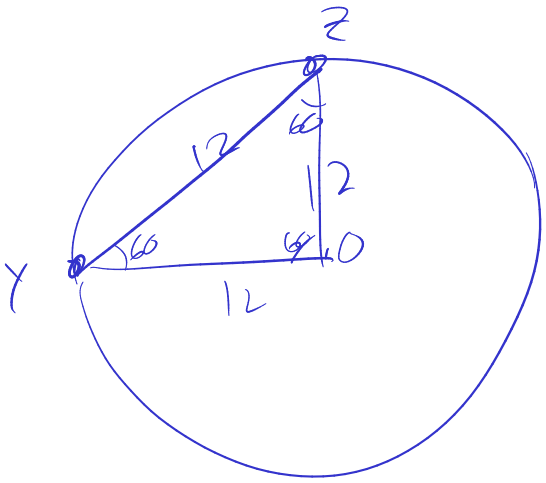
8. The length of a 72° of a circle is 15, what is the circumference of the circe?



$$15 \cdot \frac{360}{72} = 75$$

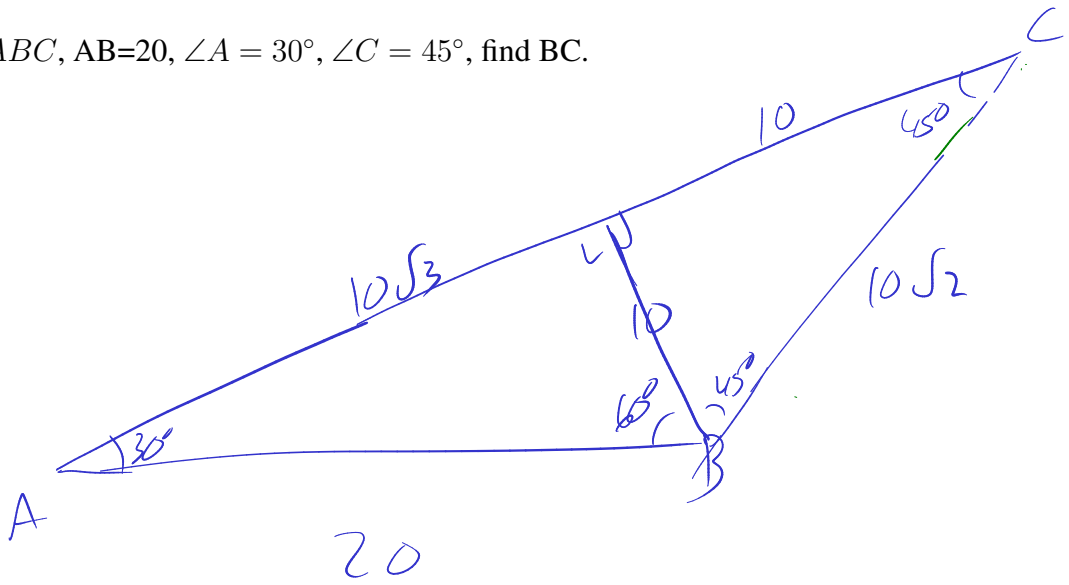
9. Chord YZ of a circle with center O has length 12. The circumference of the circle is 24π, find the length of $\overset{\frown}{YZ}$.

$= 2\pi r$

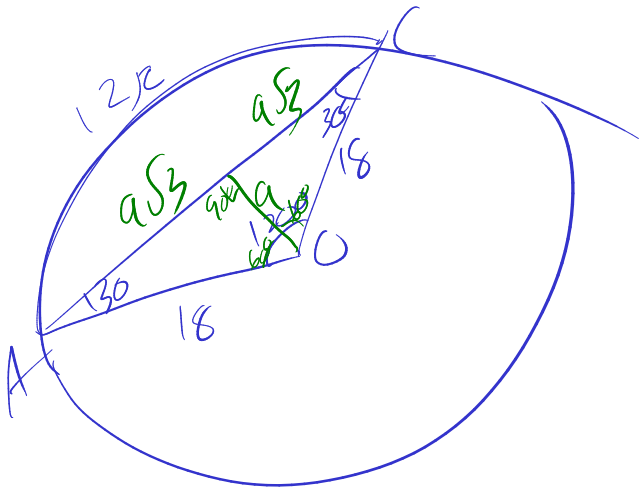


$\frac{24\pi}{6} = 4\pi$

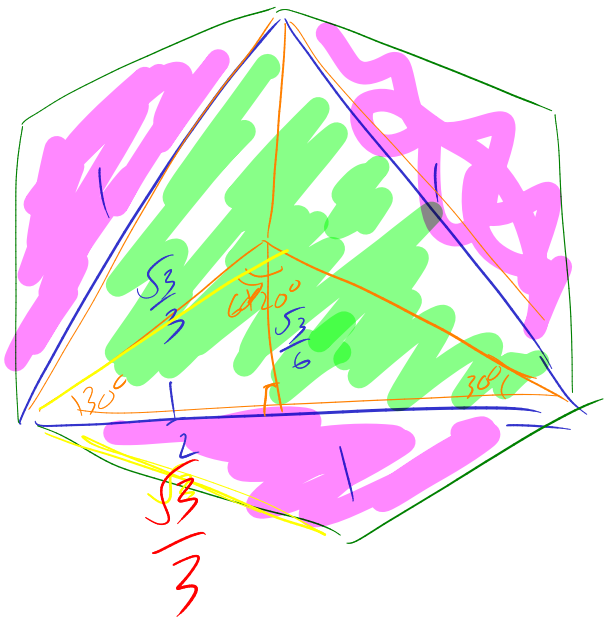
10. In $\triangle ABC$, AB=20, $\angle A = 30°$, $\angle C = 45°$, find BC.



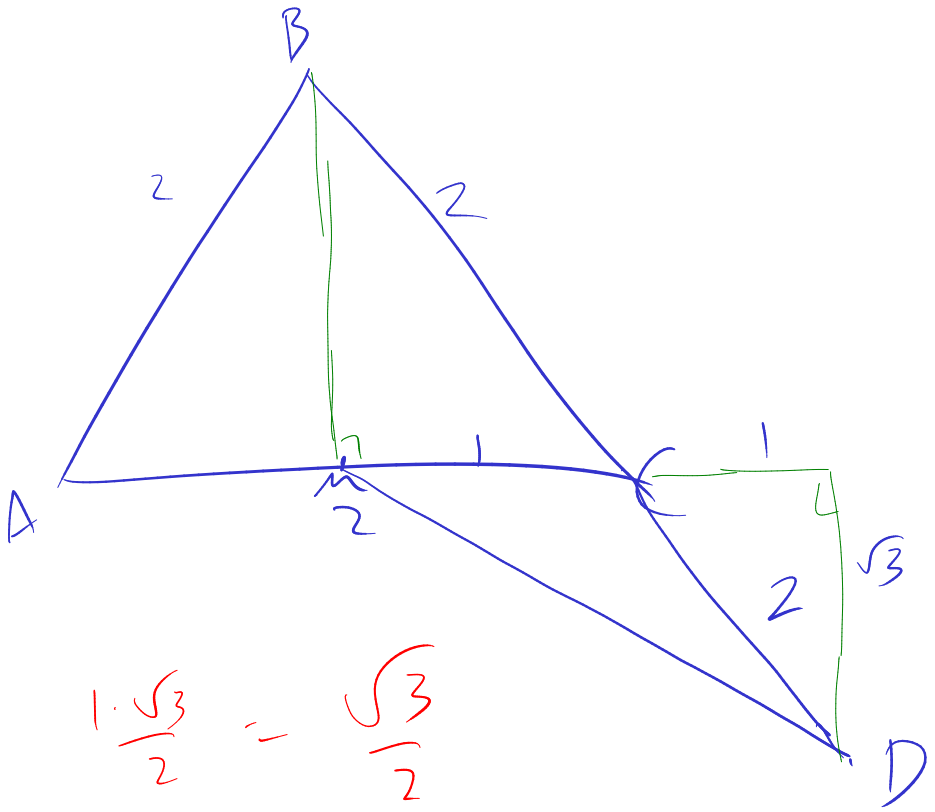
11. $\overset{\frown}{AC}$ of circle O has length 12π, the circle has radius 18. Find AC.



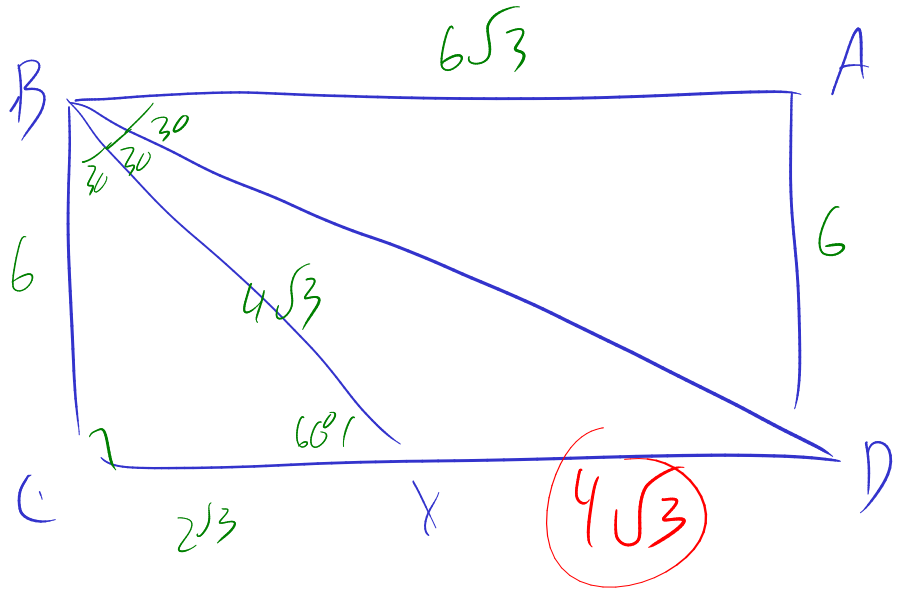
12. Three congruent isosceles trianglesare constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle, what is the length of one of the two congruent sides of one of the isosceles triangles?



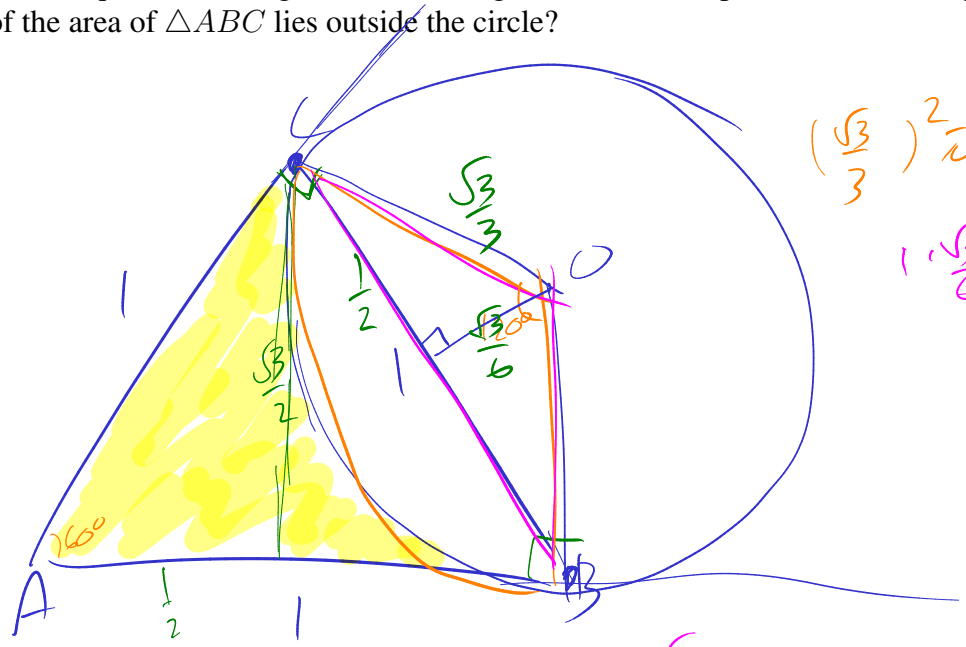
13. Equilateral triangle ABC has side length 2, M is the midpoint of AC, and C is the midpoint of BD. What is the area of $\triangle CDM$?



14. Point X is on side CD of rectangle $ABCD$ such that BX and BD trisect $\angle ABC$. If $BX = 4\sqrt{3}$, find XD.



15. Side AB and AC of equilateral triangle ABC are tangent to a circle at points B and C respectively, what fraction of the area of $\triangle ABC$ lies outside the circle?



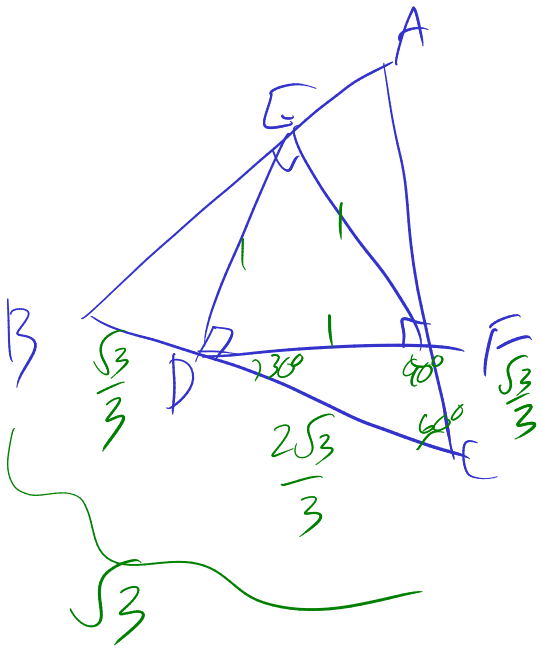
$\left(\frac{\sqrt{3}}{3}\right)^2\pi \cdot \frac{1}{3} = \frac{1}{9}\pi$

$1 \cdot \frac{\sqrt{3}}{6} \cdot \frac{1}{2} = \frac{\sqrt{3}}{12}$

$\frac{\sqrt{3}}{4} - \left(\frac{1}{9}\pi - \frac{\sqrt{3}}{12}\right)$

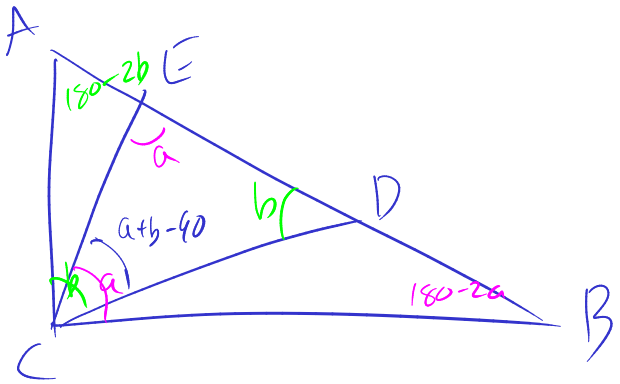
$\frac{\frac{\sqrt{3}}{3} - \frac{\pi}{9}}{\frac{\sqrt{3}}{4}} = \frac{4}{3} - \frac{4\sqrt{3}\pi}{27}$

16. Equilateral triangle DEF is inscribed in equilateral triangle ABC, such that $DEBC$. Find the ratio of the area of $\triangle DEF$ to the area of $\triangle ABC$.



$$\frac{DE}{BC} = \frac{1^2 \frac{\sqrt{3}}{4}}{\sqrt{3}^2 \frac{\sqrt{3}}{4}} = \frac{1}{3}$$

17. $\triangle ABC$ has a right angle at $\angle C$. Points D and E are on AB as shown such that $AD = AC$ and $BE = BC$. Find $\angle DCE$.



$135 - 40 = 45^\circ$

$90 + 180 - 2b + 180 - 2a = 180$

$270 = 2a + 2b$

$135 = a + b$

$a + b - 90 = 45$