

**The Ninth Grade Math Competition Class**  
**Angles, Arcs, and Special Triangles**  
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1. In the figure, given that  $\angle ABC = 60^\circ$ , and  $\angle BCD = 70^\circ$ , find  $\angle CBD$ .

2. Find  $x$  such that  $\angle APB = 2x$ ,  $\angle ACD = x$  and  $\widehat{BC} = x$ .

3. A quadrilateral is said to be cyclic quadrilateral if a circle can be drawn that passes through all four of its vertices. Prove that if ABCD is a cyclic quadrilateral, then  $\angle A + \angle C = 180^\circ$ . Such a quadrilateral is said to be inscribed in the circle.

4. The areas of two adjacent squares are 256 square inches and 16 square inches, respectively, and their bases are on the same line. What is the number of inches in the length of the segment that joins the center of the two inscribed circles?

5. We are given points A, B, C, D in the plane such that  $AD = 13$ ,  $AB = BC = AC = CD = 10$ , find  $\angle ADB$ .

6. Point  $Z$  is on side  $PR$  of  $\triangle PQR$  such that  $\angle PZQ = \angle PQZ$  and  $\angle PQR - \angle PRQ = 42^\circ$ , find  $\angle RQZ$ .

7. See the figure above and find the value of  $X$ .

8. The length of a  $72^\circ$  of a circle is 15, what is the circumference of the circle?

9. Chord  $YZ$  of a circle with center  $O$  has length 12. The circumference of the circle is  $24\pi$ , find the length of  $\widehat{YZ}$ .

**10.** In  $\triangle ABC$ ,  $AB=20$ ,  $\angle A = 30^\circ$ ,  $\angle C = 45^\circ$ , find  $BC$ .

11.  $\widehat{AC}$  of circle  $O$  has length  $12\pi$ , the circle has radius 18. Find  $AC$ .

12. Three congruent isosceles triangles are constructed with their bases on the sides of an equilateral triangle of side length 1. The sum of the areas of the three isosceles triangles is the same as the area of the equilateral triangle, what is the length of one of the two congruent sides of one of the isosceles triangles?

13. Equilateral triangle  $ABC$  has side length 2,  $M$  is the midpoint of  $AC$ , and  $C$  is the midpoint of  $BD$ . What is the area of  $\triangle CDM$ ?

14. Point  $X$  is on side  $CD$  of rectangle  $ABCD$  such that  $BX$  and  $BD$  trisect  $\angle ABC$ . If  $BX = 4\sqrt{3}$ , find  $XD$ .

15. Side  $AB$  and  $AC$  of equilateral triangle  $ABC$  are tangent to a circle at points  $B$  and  $C$  respectively, what fraction of the area of  $\triangle ABC$  lies outside the circle?

- 16.** Equilateral triangle  $DEF$  is inscribed in equilateral triangle  $ABC$ , such that  $DE \parallel BC$ . Find the ratio of the area of  $\triangle DEF$  to the area of  $\triangle ABC$ .

17.  $\triangle ABC$  has a right angle at  $\angle C$ . Points  $D$  and  $E$  are on  $AB$  as shown such that  $AD = AC$  and  $BE = BC$ . Find  $\angle DCE$ .