The Ninth Grade Math Competition Class

Congruent, Similar and Right Triangles

Anthony Wang

1. CD is the altitude from right angle $\angle ACB$ of right triangle ABC, show that $CD^2 = AD.BD$ and $AC^2 = AD.AB$.

2. If $\triangle ABC \sim \triangle XYZ$, $\frac{AB}{XY} = 4$, and $[ABC] = 64$, find $[XYZ]$.

3. Suppose $\angle ACQ = \angle QCB$, $AQ \perp CQ$, P is the midpoint of AB, show that $PQ \parallel BC$.

4. $PQ = PR$, $ZX \parallel QY$, X is on PR, Z is on the extended line of RQ, $QY \perp PR$, and PQ is extended to W such that $WZ \perp PW$, show that $\triangle QWZ \sim \triangle RXZ$, and $YQ = ZX - ZW$.

5. PA and BQ bisect angles $\angle RPQ$ and $\angle RQP$. Given $RX \perp PA$, $RY \perp BQ$, show $XY \parallel PQ$.

6. Show that if $AB \parallel CD \parallel EF$, then $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ in the diagram.

7. $TAPZ$ has $TZ \parallel AP \parallel ER$, and R, E are midpoints of AT and PZ respectively, TP and AZ intersect at point O. If $AP = 64$, $TZ = 28$, $AZ = 46$, find OI.

8. AB is divided at C such that $AC = 3CB$. Circles are drawn with AC, CB as diameters and a common tangent to these circles meets AB extended at D. Show that BD equals the radius of the smaller circle.

9. Segments AD and BE are medians of right triangle ABC and AB is its hypotenuse. If a right triangle is constructed with legs AD and BE, what will be the length of its hypotenuse in terms of AB?

10. Let ABC be an equilateral triangle and points F, Q, N satisfy $AF = QB = NC = 2AB/3$. Prove that $\angle AFQ$, $\angle NQB$, $\angle FNC$ are all 90° and FQN is an equilateral triangle.

11. The area of a a given triangle is equal to the product of an altitude and the median toward the same side. Prove that the triangle is right angled.

12. A right-angled triangle ABC is given in which F is the midpoint of the hypotenuse AB and $BC = 3AC$. Let D, E divide the side BC in 3 equal segments. Prove $\triangle DFE$ is isosceles and right angled.

13. Let M be the midpoint of side AB of equilateral triangle ABC, let N, S, K divide BC into four equal segments. P is midpoint of CM, show that $\angle MNB = \angle KPN = 90°$.