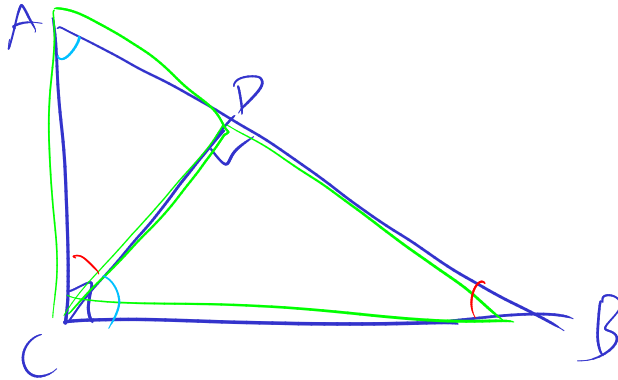


The Ninth Grade Math Competition Class

Congruent, Similar and Right Triangles

Anthony Wang

1. CD is the altitude from right angle $\angle ACB$ of right triangle ABC , show that $CD^2 = AD \cdot BD$ and $AC^2 = AD \cdot AB$.

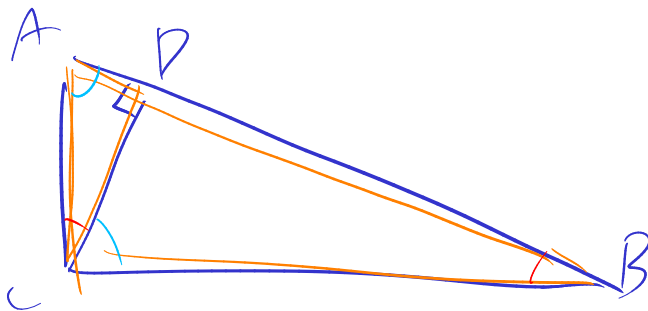


$$CD^2 = AD \cdot BD$$

$$\frac{CD}{BD} = \frac{AD}{CD}$$

$$\triangle ACD \sim \triangle CBD$$

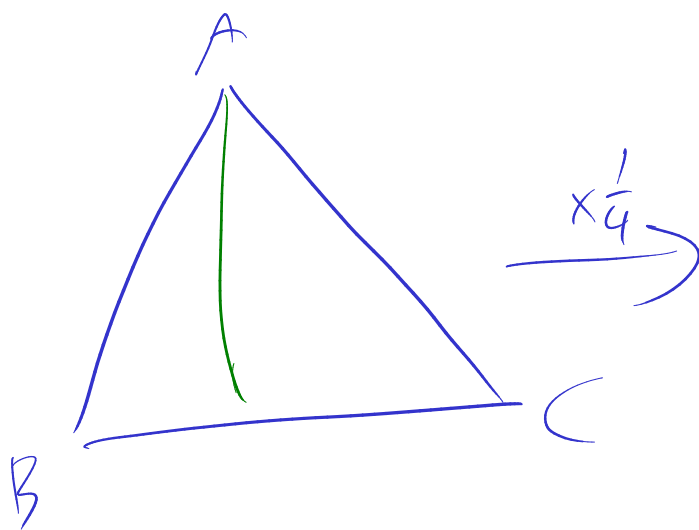
$$\frac{CD}{BD} = \frac{AD}{CD}$$



$$\triangle ACD \sim \triangle ABC$$

$$\frac{AC}{AB} = \frac{AD}{AC}$$

2. If $\triangle ABC \sim \triangle XYZ$, $\frac{AB}{XY} = 4$, and $[ABC] = 64$, find $[XYZ]$.

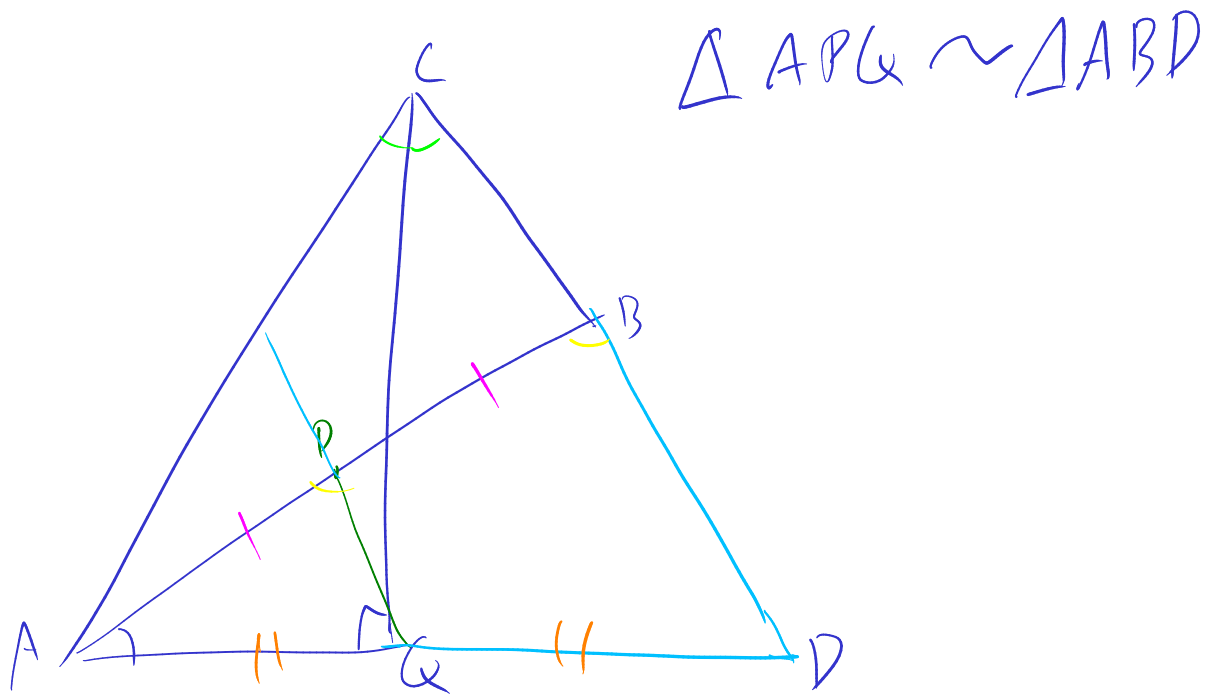


$$\frac{64}{16} = 4$$

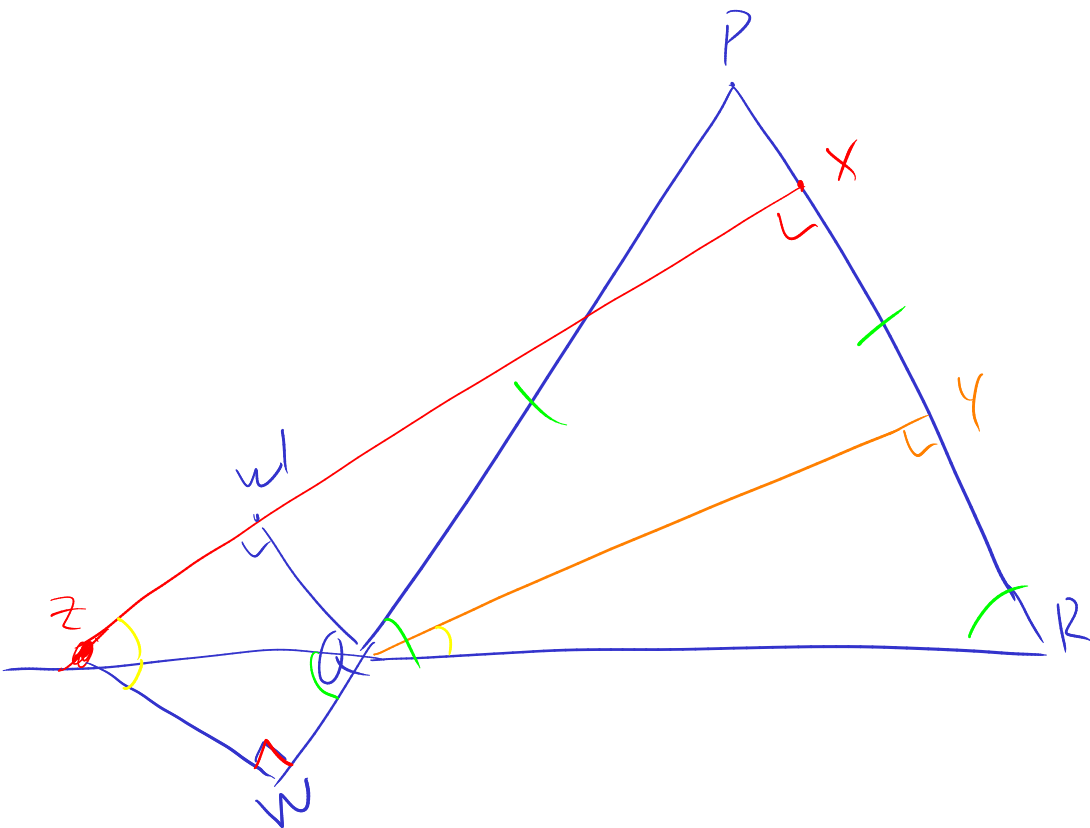
$$\begin{array}{|c|} \hline 16 \\ \hline 4 \end{array} \rightarrow \frac{16}{4}$$

length: $\frac{1}{4}$
area: $\frac{1}{16}$

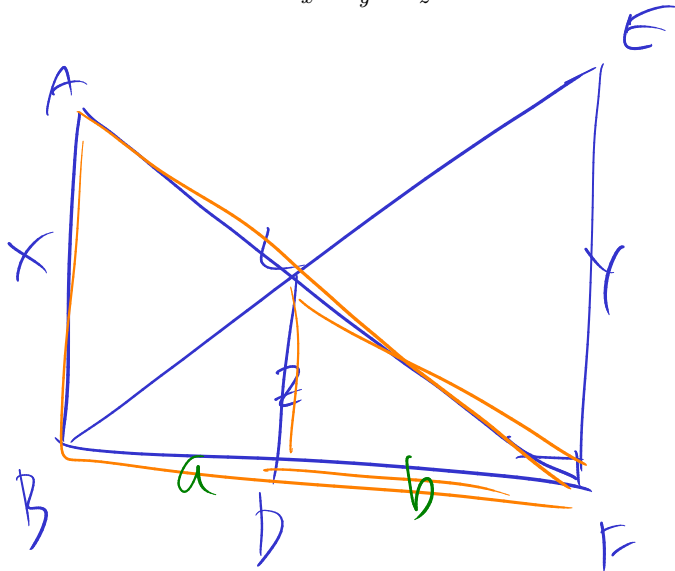
3. Suppose $\angle ACQ = \angle QCB$, $AQ \perp CQ$, P is the midpoint of AB , show that $PQ \parallel BC$.



4. $PQ = PR$, $ZX \parallel QY$, X is on PR , Z is on the extended line of RQ , $QY \perp PR$, and PQ is extended to W such that $WZ \perp PW$, show that $\triangle QWZ \sim \triangle RXZ$, and $YQ = ZX - ZW$.



6. Show that if $AB \parallel CD \parallel EF$, then $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ in the diagram.



$$\frac{x}{z} = \frac{a+b}{b}$$

$$\frac{y}{z} = \frac{a+b}{a}$$

$$\frac{z}{x} = \frac{b}{a+b}$$

$$\frac{z}{y} = \frac{a}{a+b}$$

$$\frac{z}{x} + \frac{z}{y} = 1$$

7. $TAPZ$ has $TZ \parallel AP \parallel ER$, and R, E are midpoints of AT and PZ respectively, TP and AZ intersect at point O . If $AP = 64$, $TZ = 28$, $AZ = 46$, find OI .

8. AB is divided at C such that $AC = 3CB$. Circles are drawn with AC, CB as diameters and a common tangent to these circles meets AB extended at D . Show that BD equals the radius of the smaller circle.

9. Segments AD and BE are medians of right triangle ABC and AB is its hypotenuse. If a right triangle is constructed with legs AD and BE , what will be the length of its hypotenuse in terms of AB ?

- 10.** Let ABC be an equilateral triangle and points F, Q, N satisfy $AF = QB = NC = 2AB/3$. Prove that $\angle AFQ, \angle NQB, \angle FNC$ are all 90° and FQN is an equilateral triangle.

- 11.** The area of a given triangle is equal to the product of an altitude and the median toward the same side. Prove that the triangle is right angled.

- 12.** A right-angled triangle ABC is given in which F is the midpoint of the hypotenuse AB and $BC = 3AC$. Let D, E divide the side BC in 3 equal segments. Prove $\triangle DFE$ is isosceles and right angled.

- 13.** Let M be the midpoint of side AB of equilateral triangle ABC , let N, S, K divide BC into four equal segments. P is midpoint of CM , show that $\angle MNB = \angle KPN = 90^\circ$.