The Ninth Grade Math Competition Class
Complex Numbers
Anthony Wang

1. Suppose $(-3+8i)(-3+Ai)$ is a real number, find the value of A where A is real.

2. Find all complex numbers whose squares equal $7 - 24i$.

3. Let $a = \frac{(2+i)^2}{3+i}$, find $1 + \frac{1}{a}$.

4. Find all x such that $x^5 = x^3$ (What if $x^5 = x^{-3}$).

5. If $x = \frac{1-\sqrt{3}i}{2}$, what is $\frac{1}{x^2-x}$.

6. Show that $\overline{w+z} = \bar{w} + \bar{z}$, and $\overline{wz} = \bar{w} \cdot \bar{z}$.

7. Write $\sqrt{-16+30i}$ as a complex number.

8. A function f is defined on the complex numbers by $f(z) = (a + bi)z$, where a and b are positive numbers. This function has the property that the image of each point in the complex plane is equidistant from that point and the origin. Given that $|a + bi| = 8$ and that $b^2 = \frac{m}{n}$, where m and n are positive integers, Find $m + n$.

9. There is a complex number z with imaginary part 164 and a positive integer n such that $\frac{z}{z+n} = 4i$, find n.

10. Find c if a, b, and c are positive integers which satisfy $c = (a + bi)^3 - 107i$.