Linear Equations

Ex: $x^2 + y = 4$ (2, 1, 0) → 2 — Degree

$x^2y^2 + xy = 4$ (4, 2, 0) → 4

$3^x + y = 4$ (undefined, 1, 0) → undefined

$x + y + 4z = 4$ (1, 1, 1, 0) → 1

Linear eq: Equations with degree 1

$\sqrt[3]{2x+1} - 5 + 2\sqrt[3]{2x+1} = -14$

$\sqrt[3]{2x+1} = Y$

$Y - 5 + 2Y = -14$

$3Y = -9$

$Y = -3$

$\sqrt[3]{2x+1} = -3$

$2x + 1 = -27 \Rightarrow x = -14$

Systems of Linear Equations

1. Substitution

$x + 2y = 4 \Rightarrow x = 4 - 2y = 4 - \frac{4}{5} = \frac{16}{5}$

$3x + y = 10$

$3(4 - 2y) + y = 10$

$12 - 6y + y = 10$

$y = \frac{2}{5}$

2. Elimination

$3x + 6y = 12$

$-(3x + y = 10)$

$5y = 2 \Rightarrow y = \frac{2}{5}$

$x + 2 \cdot \frac{2}{5} = 4$

$x + 2y = 4$

$3x + 6y = 12$

$0 = 0$ Infinitely many solutions

$x + 2y = 4$

$3x + 6y = 11$

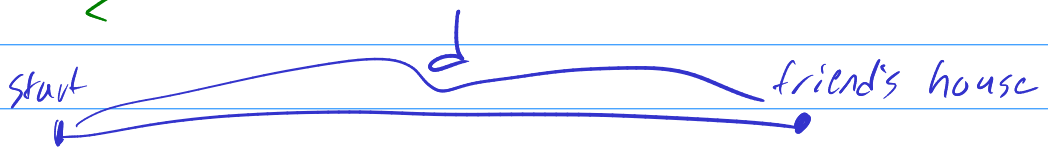
$0 = 1$ No solutions

Word Problems

Convert words to math/algebra/equations

Joshua drives 80 mph to his friend's house and 20 mph on the way back. Find his average speed for the round trip.

~~$\frac{80+20}{2} = 50$~~



$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{2d}{\frac{d}{80}+\frac{d}{20}} = \frac{2d}{\frac{20d+80d}{20\cdot 80}}$$

$$= \frac{20\cdot 80\cdot 2d}{20d+80d} = \frac{20\cdot 80\cdot 2}{100} = 32$$

Proportions

Direct: $\frac{x}{y} = c$ $\qquad x = cy$

x	y	
50¢	1 apple	$\frac{50}{1} = 50$
100¢	2 apples	$\frac{100}{2} = 50$
150¢	3 apples	$\frac{150}{3} = 50$
⋮	⋮	

Inverse

$x y = C$ (constant)

speed · time = distance

↑ ↓

↓ ↑

EX: It takes 3 days (d) for 4 people (p) to paint 5 houses (h). How long will it take for 2 people to paint 6 houses?

$dp = C_1$

$\frac{d}{h} = C_2$

$\frac{p}{h} = C_3$

$\frac{dp}{h} = C$

$\frac{d^2p^2}{h^2} = C_1 C_2 C_3$

$\frac{3 \cdot 4}{5} = C = \frac{x \cdot 2}{6}$

$x = \frac{3}{5} \cdot \frac{4}{2} \cdot \frac{6}{1} = \frac{36}{5}$

$k = \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d} = \frac{bk+dk}{b+d} = \frac{(b+d)k}{b+d} = k$

$a = bk$

$c = dk$

$\frac{a-c}{b-d} = \frac{bk-dk}{b-d} = k$

$$\frac{ma+nc}{mb+nd} = \frac{bkm+dkn}{mb+nd} = k$$

Ex: $\frac{x+2y}{x-y} = \frac{3}{4}$ Find $\frac{x}{y}$

$4x+8y = 3x-3y$

$x = -11y$

$\frac{x}{y} = -11$