Prime Factorizations

Prime number: Integer with 2 divisors:
1 and itself

7, 2, 3, 5

Composite: Integers that aren't prime

Is 87 prime? No $\quad 87 = \underset{>\sqrt{n}}{\overset{<\sqrt{n}}{3 \cdot 29}}$

does not divide

$2 \nmid 87 \Rightarrow 4 \nmid 87$ $\quad 100 = 10 \cdot 10$

$3 \mid 87$

divides

To check if n is prime:
check primes up to $\sqrt{n}$

Is 209 prime?

$\sqrt{209} = 14.4\ldots$

2, 3, 5, 7, 11, 13

$209 = 11 \cdot 19$ $\quad$ 209 is composite

Ex: Find the smallest composite # with no prime factors less than 10

~~1~~ 2 3 5 7

11 13 17 . . . —

$11 \cdot 13 = 143$

$11 \cdot 11 = 121$

$$\begin{aligned} 120 &= 2 \cdot 60 \\ &= 2 \cdot 2 \cdot 30 \\ &= 2 \cdot 2 \cdot 2 \cdot 15 \\ &= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \\ &= 2^3 \cdot 3^1 \cdot 5^1 \end{aligned}$$

Prime factorization

GCD: Greatest common divisor

12, 45

Divisors: 12: 1, 2, 3, 4, 6, 12
45: 1, 3, 5, 9, 15, 45

$\gcd(12, 45) = 2^0 \cdot 3^1 \cdot 5^0 = 3$

$12 = 2 \cdot 6 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3^1 \cdot 5^0$

$45 = 2^0 \cdot 3^2 \cdot 5^1$

LCM: Least common multiple

$\mathrm{lcm}(12,45) = 180$

$12 = 2^2 \cdot 3^1 \cdot 5^0$

$45 = 2^0 \cdot 3^2 \cdot 5^1$

$180 = 2^2 \cdot 3^2 \cdot 5^1$

$\gcd(12,45) \cdot \mathrm{lcm}(12,45) = 3 \cdot 180$

$= 540$

$12 \cdot 45 = 540$

$\gcd(a,b) \cdot \mathrm{lcm}(a,b) = ab$

$\gcd(800, 1800) = 200$

$\gcd(8,18) = 2$

$\gcd(ac, bc) = c \cdot \gcd(a,b)$

$$\operatorname{lcm}(ac, bc) = c \cdot \operatorname{lcm}(a,b)$$

Divisors

How many divisors does n have?

$24 = 1, 2, 3, 4, 6, 8, 12, 24$

8 divisors

$24 = 2^3 \cdot 3^1$

	2^0	2^1	2^2	2^3
3^0	1	2	4	8
3^1	3	6	12	24

$24 = 2^3 \cdot 3^1$

2^0 3^0

2^1 3^1

2^2

2^3

$4 \cdot 2 = 8$

$$120 = 2^3 \cdot 3^1 \cdot 5^1$$

$$\begin{array}{ccc} 2^0 & 3^0 & 5^0 \\ 2^1 & 3^1 & 5^1 \\ 2^2 & & \\ 2^3 & & \end{array}$$

$$(3+1)(1+1)(1+1) = 16$$

$$5400 = 54 \cdot 100$$
$$= 2^1 \cdot 3^3 \cdot 2^2 \cdot 5^2 = 2^3 \cdot 3^2 \cdot 5^2$$

$$\underset{4}{(3+1)}\underset{3}{(2+1)}\underset{3}{(2+1)} = 36$$

$d \mid 5400$

$$d = \begin{array}{ccc} 2^0 & 3^0 & 5^0 \\ 2^1 & 3^1 & 5^1 \\ 2^2 & 3^2 & 5^2 \\ 2^3 & & \\ \cancel{2^4} & & \end{array}$$

$t(n)$ = total # of divisors of n

$$n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$$
$$t(n) = (e_1+1)(e_2+1) \cdots (e_k+1)$$

$m = 2^3 \cdot 3^2 = 72$ $\quad mn = 2^3 \cdot 3^2 \cdot 5^2 = 1800$

$n = 5^2 = 25$

$a = 2^1 \cdot 5^2 \quad t(a) = (1+1)(2+1)$

$ma = 2^4 \cdot 3^2 \cdot 5^2 \quad t(ma) = (4+1)(2+1)(2+1)$

$t(m) = (3+1)(2+1) = 12$

$t(n) = (2+1) = 3$

$t(mn) = (3+1)(2+1)(2+1) = 36$

$t(m)t(n) = t(mn)$

This only works if $\gcd(m,n) = 1$